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Enclosure 1

Final Technical Report on W911NF-05-1-0474
A new class of materials for quantum information processing

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I. INTRODUCTION

The materials described by Landau symmetry breaking theory have had enormous impact on technology. Ferromagnetic materials that break spin rotation symmetry can be used as the media of digital information storage. A hard drive made by ferromagnetic materials can store so much information that a whole library can be put in it. Liquid crystals that break rotation symmetry of molecules find wide application in display. Nowadays one can hardly find a household without liquid crystal display somewhere in it. Crystals that break translation symmetry lead to well defined electronic band which in turn allow us to make semiconducting devices. Semiconducting devices make the high tech revolution possible which changes the way we live.

However, recent studies show that there exist new classes of states of matter that cannot be described by Landau symmetry breaking theory.[1] String-net condensed states[2] are one of the those new classes of materials, which are even richer than the old symmetry breaking states. After seeing so much impact of symmetry breaking states, one cannot help to imagine the possible applications of the richer string-net condensed states.

One possible applications is to use string-net condensed states as media for quantum computing. String-net condensed state is a state with complicated quantum entanglement. As a many-body system, the quantum entanglement in string-net condensed state is distributed among many different particles/spins. As a result, the pattern of quantum entanglements cannot be destroyed by local perturbations. This significantly reduces the effect of decoherence. If we use different quantum entanglements in string-net condensed state to encode quantum information, the quantum information can last for a long time.[3] So we can use string-net condensed state as quantum memory.

The quantum information encoded by the string-net entanglements can also be manipulated by dragging the ends of strings around each others. This process realizes quantum computation[4], or more generally, quantum information processing. It was shown theoretically that certain string-net condensed states can realize arbitrary quantum information processing.[4] So those string-net condensed states are realizations of the universal quantum computer. We see that string-net condensed states are natural media for both quantum memory and quantum computation. Such realizations of quantum memory and quantum computation are naturally fault tolerant.[5]

However, right now, we only know how to construct theoretical models that give rise to string-net condensations. The next step is to design realistic materials or to find realistic materials that have string-net condensations. This will be the proposed research of this project. In this project we develop a mean-field approach for string condensed states. This will help us to determine which experimental systems are most suitable for realizing the theoretical models that contain string-net condensations.

II. A MEAN-FIELD APPROACH FOR STRING CONDENSED STATES

Recently, several frustrated spin systems have been discovered with the unusual property that their collective excitations are described by Maxwell's equations. [6–11] These light-like collective modes can be traced to the highly entangled nature of the ground state. In these systems, the low energy degrees of freedom are not individual spins, but rather string-like loops of spins. The ground state is a coherent superposition of many such string-like configurations - a string condensate discussed above. It is this "string condensation" in the ground state that is responsible for the emergent photon - just as particle condensation is responsible for the phonon modes in a superfluid. [2, 8, 12, 13]

While this qualitative picture is relatively clear, quantitative results on string condensation and artificial light are lacking. The above models have only been analyzed in limiting (and unrealistic) cases. Current theoretical result still cannot help us to identify realistic systems with string-net condensation. The problem is that we are missing a good mean field approach for string-net condensed states. The conventional mean field theory approaches can only be applied to symmetry breaking states with local order parameters. They are useless for understanding string condensed states which are highly entangled and have nothing to do with symmetry breaking.

In this project,[14] we address this problem. We describe a mean field approach that can be applied to both symmetry breaking *and* string condensed states. We hope that this technique can be used to identify conditions under which string condensation may occur, and to help further the experimental search for emergent photons and new states of matter with string-net condensation.

In practice, our approach can be thought of as a mean field technique for quantum string (or dimer) models. This technique can be used to estimate the phase diagram of string (or dimer) models, to find the low energy dynamics of the different phases, and to analyze the phase transitions. It can be applied to any quantum spin system with the property that its low energy degrees of freedom are strings or dimers. This includes all the frustrated spin systems cited above.

Our meanfield approach is a variational approach – The variational wave functions Ψ have a large number of variational parameters $\{z_{\boldsymbol{ij}}\}$ indexed by the oriented links $\boldsymbol{ij}$ of the lattice. For each set of $\{z_{\boldsymbol{ij}}\}$, the corresponding wave function $\Psi_{\{z\}}$ is defined by

$$\Psi_{\{z\}}(X) = \prod_{\boldsymbol{ij}} z_{\boldsymbol{ij}}^{n_{\boldsymbol{ij}}} \tag{II.1}$$

where $n_{\boldsymbol{ij}}$ is the occupation number of the oriented link $\boldsymbol{ij}$ in the oriented string configuration X. $n_{\boldsymbol{ij}} = 0$ implies that the link $\boldsymbol{ij}$ is not occupied by strings. The above variational wave function (II.1) can accommodate many different kinds of states - including both string crystals and string liquids. If $z_{\boldsymbol{ij}}$ is periodic, $\Psi_{\{z\}}$ is a symmetry breaking string crystal state (which correspond to various spin ordered states); if $z_{\boldsymbol{ij}}$ is constant for all $\boldsymbol{ij}$, then $\Psi_{\{z\}}$ is a string liquid state. The variational states $\Psi_{\{z\}}$ can even access the two *types* of string liquids described in the previous section - small string states (the normal state) for small $|z_{\boldsymbol{ij}}|$ and string condensed states for $|z_{\boldsymbol{ij}}| \approx 1$.

We demonstrate the technique with a simple example: a spin-1 XXZ model on the Kagome lattice [8]:

$$H = J_1 \sum_{\boldsymbol{I}} (S^z_{\boldsymbol{I}})^2 + J_2 \sum_{\langle \boldsymbol{IJ} \rangle} S^z_{\boldsymbol{I}} S^z_{\boldsymbol{J}} - J_{xy} \sum_{\langle \boldsymbol{IJ} \rangle} (S^x_{\boldsymbol{I}} S^x_{\boldsymbol{J}} + S^y_{\boldsymbol{I}} S^y_{\boldsymbol{J}}) \tag{II.2}$$

Here $\boldsymbol{I}$ and $\boldsymbol{J}$ label the sites of the Kagome lattice, and $\sum_{\langle \boldsymbol{IJ} \rangle}$ sums over all nearest neighbor sites. This model provides a good testing ground for the method since the low energy dynamics of H is described by a string model in the regime $J_2 \gg J_{xy} \gg |J_1 - J_2|$.

The mean field calculation predicts a number of interesting phases including string condensed phases with emergent photons. The string condensed phases are ultimately destroyed once instanton fluctuations are included, but several phases and phase transitions remain - including a quantum critical point with emergent photons that have a $\omega \propto k^2$ dispersion.

The mean field phase diagram for (II.2) is shown in Fig. 1a. Here, $J = \frac{9J_{xy}^2}{J_2} + \frac{24J_{xy}^3}{J_2^2} + 3(J_1 - J_2)$, and $g = \frac{3J_{xy}^3}{J_2^2}$. For large positive J/g, the system is in a paramagnetic phase, with no broken symmetries while for large negative J/g, the system is in a plaquette ordered phase with broken

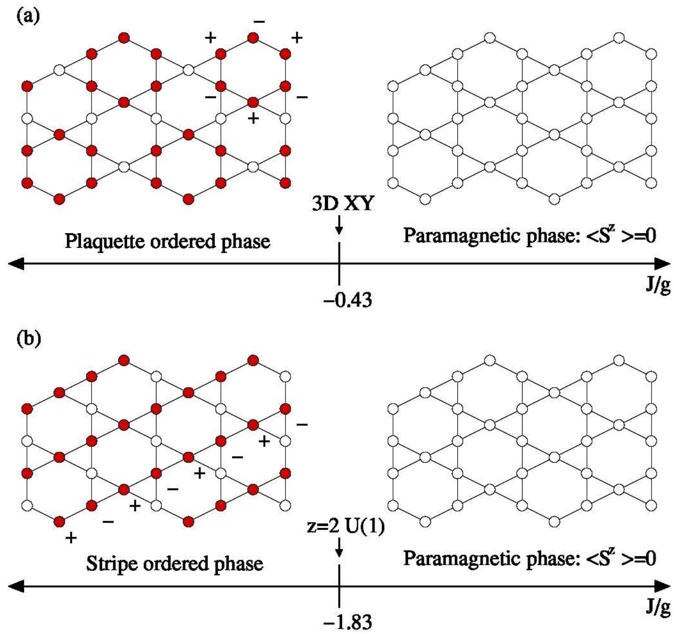

FIG. 1: The mean field phase diagram for (a) the XXZ model (II.2) and (b) the XXZ model with additional next nearest neighbor interactions. The filled circles denote spins with $\langle S^z \rangle \neq 0$. The sign of $\langle S^z \rangle$ alternates around each plaquette in the plaquette ordered phase and along each stripe in the stripe ordered phase.

lattice and spin symmetries. The critical point is in the universality class of the 3D XY model (see also Ref. [15]).

We also study the model (II.2) with an additional second nearest neighbor interaction $J_3 \sum_{\langle\langle ij \rangle\rangle} S^z_i S^z_j$, $J_3/g = 0.17$. We find a different phase diagram (Fig. 1b). For large positive J/g, the system is in a paramagnetic phase, while for large negative J/g the system is in a stripe ordered phase with broken rotational and spin symmetry. The mean field calculation predicts that the phase transition is a deconfined quantum critical point described by a $z = 2$ $U(1)$ gauge theory.

III. DETECTING STRING-NET CONDENSATION THROUGH TOPOLOGICAL ENTANGLEMENT ENTROPY

Until recently, the only known physical characterizations of topological order [1] in the string-net condensed states involved properties of the Hamiltonian - e.g. quasiparticle statistics [16], ground state degeneracy [17, 18], and edge excitations [1]. In this project,[19] we demonstrate that topological order is manifest not only in these dynamical properties but also in the basic entanglement of the ground state wave function. We hope that this characterization of topological order can be used as a theoretical tool to classify trial wave functions - such as resonating dimer wave functions [20], Gutzwiller projected states, [21–25] or quantum loop gas wave functions [26]. In addition, it may be useful as a numerical test for topological order. Finally, it demonstrates definitively that topological order is a property of a wave function, not a Hamiltonian. The classification of topologically ordered states is nothing but a classification of complex functions of thermodynamically large numbers of variables.

Let us consider $(2+1)$ dimensional systems (though the result can be generalized to any dimension). Let Ψ be an arbitrary wave function for some two dimensional lattice model. For any subset A of the lattice, one can compute the associated quantum entanglement entropy S_A.[27] The main result of this paper is that one can determine the "total quantum dimension" D of Ψ by computing the entanglement entropy S_A of particular regions A in the plane.

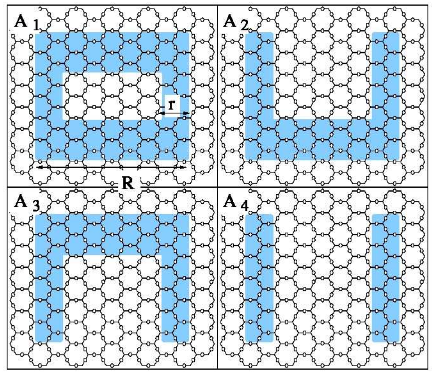

FIG. 2: One can detect topological order in a state Ψ by computing the von Neumann entanglement entropies S_1, S_2, S_3, S_4 of the above four regions, A_1, A_2, A_3, A_4, and then taking the linear combination $(S_1 - S_2) - (S_3 - S_4)$ in the limit of $R, r \to \infty$. Here the four regions are drawn in the case of the honeycomb lattice. Note that these regions have been carefully designed so that A_1 differs from A_2 in the same way that A_3 differs from A_4.

Normal states have $D = 1$ while topologically ordered states have $D > 1$. Thus, this result provides a way to distinguish topologically ordered states from normal states, *using only the wave function.*

More specifically, consider the four regions A_1, A_2, A_3, A_4 drawn in Fig. 2. Let the corresponding entanglement entropies be S_1, S_2, S_3, S_4. We find that the linear combination $(S_1 - S_2) - (S_3 - S_4)$, in the limit of large, thick annuli, $R, r \to \infty$, is a universal number that is robust against any local perturbations. Furthermore

$$(S_1 - S_2) - (S_3 - S_4) = -\log(D^2) \tag{III.1}$$

where D is the total quantum dimension of the topological order associated with Ψ. We call the quantity $(S_1 - S_2) - (S_3 - S_4)$ the "topological entropy", $-S_{\text{top}}$, since it measures the entropy associated with the (non-local) topological entanglement in Ψ. S_{top} only depends on the type of topological order associated with Ψ.

IV. SUPPORTED STUDENT

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